Optimization Algorithm Applied to the Design of Few-Mode Erbium Doped Fiber Amplifier for Modal and Spectral Gain Equalization

Guillaume Le Cocq, Yves Quiquempois, and Laurent Bigot

***Abstract*—Gradient descent optimization algorithm is applied to design few-mode Er^{3+}-doped fibers adapted to gain equalization over modes and wavelengths, for mode division multiplexing. Theoretical study is performed for fiber designs supporting six and ten spatial modes at signal wavelength. Flat gain is obtained by optimizing Er^{3+} doping profile of a micro-structured core and modal composition of the pump beam.**

***Index Terms*—Erbium-doped fiber amplifiers (EDFA), gradient methods, multiplexing.**

I. Introduction

Space division multiplexing has been widely investigated during the last few years as a new technique that could permit to increase the capacity of optical fibers [1]–[6]. Today, two technologies are deeply considered, namely multicore fibers (MCF) and few-mode fibers (FMF). In the case of FMF, each transverse mode can be used as an individual pathway able to carry the total data transmission capacity of a current single-mode fiber (SMF). However, so as to make this technology compatible with long-haul transmissions, periodic amplification of the signals, multiplexed on the different modes and the different wavelengths, will be necessary. In this paper, a study on few-mode erbium doped fiber amplifier (FM-EDFA) especially designed for mode division multiplexed (MDM) transmissions is reported. The role of such amplifiers is to provide high and flat gain for the different channels that are used for data transmission (meaning the same gain over wavelengths and modes). Different techniques can be used to reach such gain equalization: 1) tailoring the Er^{3+} doping profile (EDP) [7]–[11], 2) tailoring the pump intensity pattern [11]–[13], and 3) concatenating different FM-EDF [14].

Previous works reported good performances with few-mode erbium doped fibers (FM-EDF) made by modified chemical vapor deposition (MCVD) process [8], [11]. However, this fabrication process seems to be quite limiting if one needs to realize EDP including complex shaping, especially when combined with solution doping. Diffusion of the Er^{3+} ions during preform and fiber fabrication partly explains this difficulty [15]. We recently proposed to use micro-structured core fibers to more accurately control EDP, and hence, gain equalization between modes [9]. This last technique appears as an interesting alternative to MCVD because it allows to finely tailor the EDP in the fiber core, without significant impact from Er^{3+} diffusion. However, whatever the technique used to manufacture the fiber, the optimization of the Er^{3+} distribution and pump beam composition is a complex multi-parameters problem.

In this paper, a numerical optimization approach is combined to a multimode amplification code in order to identify amplifier designs allowing gain equalization over C-band and over the different modes supported by a micro-structured core fiber. A special attention has also been paid to reduce the pump power consumption in order to increase the power efficiency of the FM-EDFA, which is a very important issue to make MDM technology competitive. Thus, all the optimizations that will be presented in this paper, have been performed with a limited pump power budget of 200 mW. Moreover, pump insertion losses have been included in the calculations. In the frame of this paper, the modes will be considered as linearly polarized (LP) modes and their confinement losses will be neglected as they can be made as low as suitable by increasing the number of rings.

Both Er^{3+} distribution and pump intensity pattern are considered for optimization. The study is first focused on developing an optimized design of an Er^{3+}-doped fiber that supports six spatial modes [namely LP_{01}, LP_{11a}, LP_{11b}, LP_{02}, LP_{21a}, LP_{21b} represented in Fig. 1(b)] at 1550 nm and allows low gain excursion (ΔG) and high average gain (G_{ave}). Similarly to the concept recently reported [9], micro-structured core fiber geometry is studied in this paper: the core of the fiber is composed of 19 elements, each of them having a particular erbium concentration but presenting the same refractive index than the others. In the present case, the micro-structured core is combined to an air/silica cladding [see Fig. 1(a)] but one should note that the concept of micro-structured core can be extended to all-solid fibers. A proof of feasibility of such a fiber design has recently been demonstrated [9]. Micro-structuring the fiber core opens a lot of possibilities since we consider that the Er^{3+} concentration of each rod that composes the core constitutes a possible degree of freedom (DoF). However, it seems not straightforward

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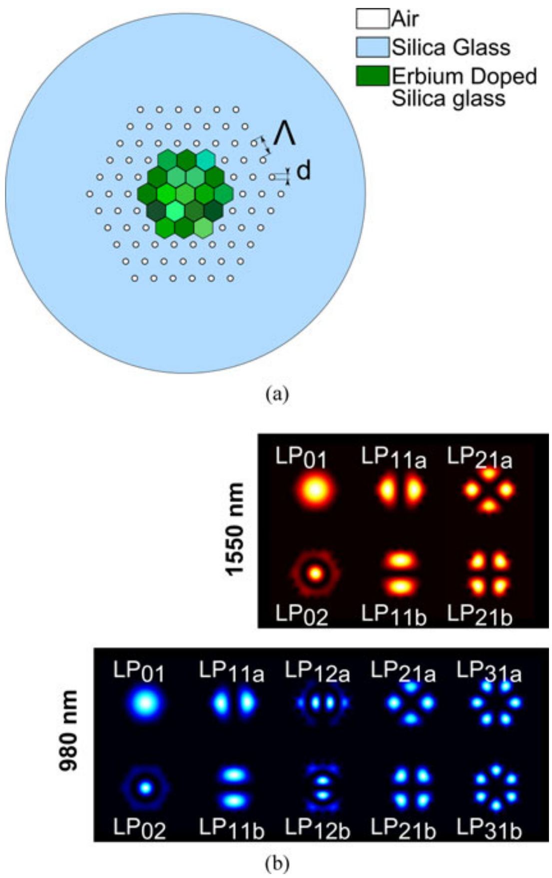

Fig. 1. (a) Example of fiber geometry considered in this paper: silica capillary, pure silica glass and Er^{3+} -doped rods are stacked together, and then, drawn into a fiber with micro-structured core and air/silica cladding. (b) Calculated intensity profiles of the guided modes at 1550 and 980 nm, calculated by using COMSOL Multiphysics.

to reach an optimized design with a simple parametric approach. In order to find the best EDP that is achievable with the 19 elements of the core, computations were made by using an home-made code used to obtain the modal gain in FM-EDFA [8], [12] and this code has been combined to a gradient descent optimization algorithm (GDOA). This technique is well adapted to the case of multi-parameters problems [16], [17]. Note that in a similar manner, a recent work also proposed to use another inverse method (genetic algorithm) to tailor the erbium distribution in the case of a cladding-pumped few-mode EDFA [18], [19] made by the conventional MCVD method. The background theory is first presented and the results are discussed with a specific focus on the influence of the pumping scheme. The approach is then extended to a ten-mode amplifier considering a ten dimensions problem.

II. GDOA

GDOA can be used to greatly reduce computation time necessary to find the minimum (or the maximum) of a function ($F(\vec{\mathbf{X}})$) with numerous variables (that can be considered as a vector $\vec{\mathbf{X}} = [X_1\ X_2\ X_3\ X_4\ ...\ X_n]$). Starting from an initial point $\vec{\mathbf{X}}_0$, the algorithm will obey to a step by step process following this function:

$$\vec{\mathbf{X}}_{i+1} = \vec{\mathbf{X}}_i + \gamma \times \vec{\mathbf{grad}}\{F(\vec{\mathbf{X}})\}$$

where γ is a step size factor and i represents the number of iterations. The sign of γ will determine if the algorithm will converge either to the maximum or the minimum of the function (respectively, positive and negative sign). γ can be a constant or a varying parameter if adaptive step size is required. For each iteration, the vector $\vec{\mathbf{X}}$ becomes closer and closer to the solution ($\vec{\mathbf{X}}_s$). The drawback of such a method is that the algorithm can converge to a local extremum and not to the global one. However, this can be circumvent by choosing different starting points and comparing the different solutions that are obtained in order to find the best solution.

In this study, the GDOA algorithm will be used to find the EDP that will permit to reach the best performances: the function F will represent the amplifier performances and $\vec{\mathbf{X}}$ will correspond to the EDP. Before all, one has to find a good definition of the amplifier performances, i.e., high average gain (G_{ave}) and low gain excursion (ΔG). Note that other targets are possible, as reducing the DMG over the whole C-band, or obtaining higher gain on particular higher order modes that suffer of higher confinement losses in the fiber line. After some tests, it has been considered that a good tradeoff is to consider the function F as

$$F(\vec{\mathbf{X}}) = G_{\text{ave}}(\vec{\mathbf{X}}) - 10 \times \Delta G(\vec{\mathbf{X}})$$

because it linearly changes with gain values (which will help the GDOA to converge). The factor 10 before ΔG implies quick convergence toward an EDP that allows low ΔG, followed by a slower convergence to an EDP that allows high gain. To do so, each term of $\vec{\mathbf{X}}$ represents the erbium concentration of the ith doped rod of the core:

$$\vec{\mathbf{X}} = [C_1\ C_2\ C_3\ C_4\ ...\ C_n]$$

so that each rod concentration becomes a DoF. As the core is composed of 19 elements, 19 DoF can be used for GDOA simulations—even if it will be proved later in this paper that 19 variables is over-dimensioned for the physical problem considered here. Thus, by using GDOA, the EDP slightly evolves so that the value of $F(\vec{\mathbf{X}})$ increases (step by step), and as a consequence, G_{ave} increases and ΔG decreases.

In the first part of this paper, 24 signals (6 modes $\times$ 4 wavelengths) simultaneously amplified are considered. The signals are distributed over the C-band (at 1530, 1540, 1550, and 1560 nm) and over the six guided modes (namely LP_{01}, LP_{11a}, LP_{11b}, LP_{21a}, LP_{21b}, and LP_{02}). The opto-geometrical parameters of the photonic crystal fiber (PCF) have been chosen so that the waist of the fundamental mode around 1550 nm is close to the one of the passive FMF used in our experiments at the input/output of the FM-EDF [20]. This leads to an air/silica cladding with $\Lambda = 3.3$ μm and d/$\Lambda = 0.3$. The guided modes of this PCF have been calculated at 1550 (signal wavelength) and 980 nm (pump wavelength) by using a finite-element method. The intensity profiles of these modes are reported in Fig. 1(b). Each signal has an input power around -18 dBm so that the total input signal power is -4 dBm. In this first part, pump beam is

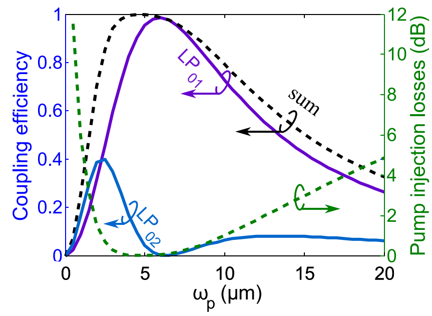

Fig. 2. Coupling efficiency between guided modes and incident Gaussian beam as a function of injection waist ω_p at 980 nm. Incident pump beam is considered to be injected in centered conditions. The green dashed curve represents pump injection losses that are taken into account in the calculations as a function of ω_p, so that, in the amplification code, the incident power is reduced to the real pump power coupled in the fiber.

considered to be injected in free space at the center of the fiber core so that only LP_{0m} pump modes are excited at 980 nm, as in reference [8] and [9]. More complex pumping schemes are possible by using phase plates as pump mode converters [11], [12] or by using off-centered pump injection conditions so that several pump modes are excited [13]: these schemes will be considered at the end of this paper. Even if these advanced pumping schemes offer a large range of possibilities, we consider, at first, that a simple centered pump injection conditions posseses some intrinsic advantages like 1) low coupling losses, 2) pump intensity pattern with cylindrical symmetry, and 3) easy adjustments. Incident pump power (P_p) is fixed to 200 mW in the following simulations (this pump power value is a good tradeoff between performances and power efficiency). The waist of the injected Gaussian pump beam (ω_p) is considered as a new DoF for optimization because it can be adapted so as to change the ratio of optical power coupled in each LP_{0m} mode at 980 nm (with respect to the overlap integrals between the guided modes at pump wavelength and the incident pump Gaussian beam [8]) (see Fig. 2). Note that considering such a pumping scheme implies some insertion losses due to mode profile mismatch. These losses will be included in our calculations in the following of the paper. As can be seen in Fig. 2, insertion losses remain very low, since they are always below 2 dB for all ω_p between 2 and 14 μm. GDOA has been used to optimize the EDP so as to find the one that gives the lowest ΔG and the highest G_{ave} between the 24 channels. The Er^{3+} concentration in the ith rod family (C_i) has been considered to be between 0 and 8×10^{24} ions.m^{-3}. For each design and each iteration of the optimization, the fiber length has been adjusted to the value that minimizes ΔG and maximizes average gain, by using a second optimization tool (simple parametric approach).

Besides the need to reduce computation time and propose realistic designs in view of manufacturing considerations, the number of DoF has been reduced from 19 to 4, for the following reasons.

1) The number of DoF can be reduced from 19 to 7 because the guided modes at signal wavelength present rotational symmetry (or quasi rotational—in fact C_{6v} due to the hexagonal cladding—symmetry) of order 2 and 4 for LP_{11} and LP_{21} modes and ∞ for LP_{01} and LP_{02} modes. It is the same at pump wavelength: LP_{01} and LP_{02} modes have both a quasi rotational symmetry of order ∞. So, if the core elements are grouped by family of one or three rods of same concentration distributed with a rotational symmetry of order 3 [so that each element is the apex of an equilateral triangle, as in Fig. 3(b)], the resulting EDP will be more simple, without changing the modal gain of the amplifier.
2) The number of DoF can be finally reduced from 7 to 4 because it appeared that the performances of the amplifier only depend on four variables, namely C_1, $(C_2 + C_3)$, $(C_4 + C_6)$, and $(C_5 + C_7)$. Thus, the core elements are grouped in new families, as a function of the distance between the center of the rods and the center of the core, as is represented in Fig. 3(c).

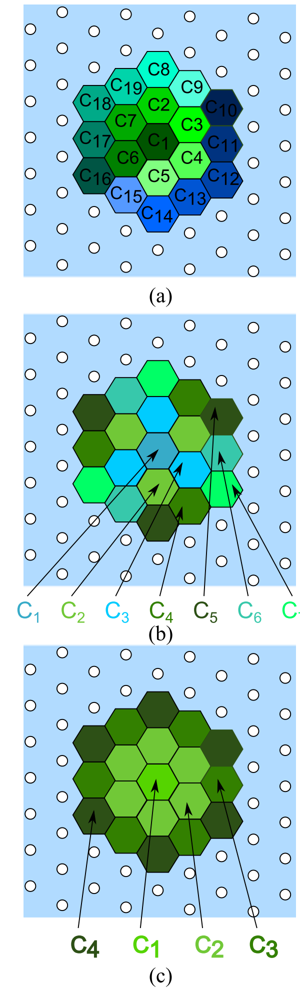

Fig. 3. (a) Schematic representation of EDP with the 19 rods that constitute 19 DoF of the problem. (b) EDP when the number of DoF is reduced to 7 (due to rotational symmetry considerations). c) EDP when the number of DoF is reduced to 4.

III. EDP Optimization—Study of Convergence

In this section, ω_p will be considered as a constant, in order to analyze the convergence of the EDP, when using GDOA with

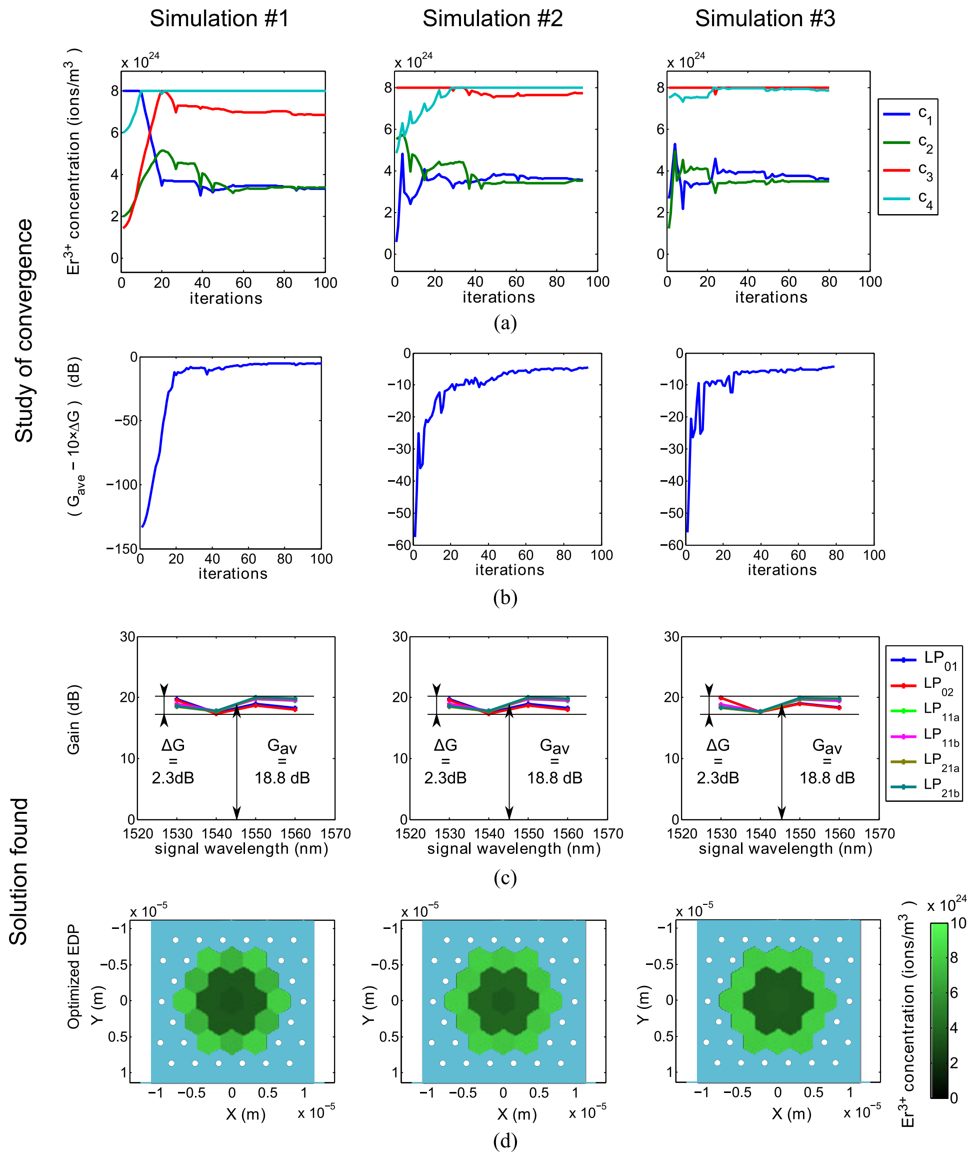

Fig. 4. (a) Evolution of the EDP (Er^{3+} concentration of each core elements) as a function of the number of GDOA iterations. (b) Amplifier performances (defined by function F) as a function of the number of GDOA iterations. (c) Gain as a function of signal mode and signal wavelength, when EDP is equal to the solution found by the GDOA. d) 2-D representation of the EDP that is the solution found by the GDOA [i.e., final values of Erbium concentrations reported in Fig. 4 (a)].

fixed pumping scheme. ω_p has been fixed to 2.5 μm so that LP_{01} and LP_{02} pump modes are equally excited (see Fig. 2), and pump injection losses are only about 0.7 dB (meaning that 85% of the 200 mW pump power are coupled into the fiber core). Three examples of GDOA simulations are reported in Fig. 4. For each simulation, the starting point ($\vec{\mathbf{X}}_0$) is a random vector.

In Fig. 4, each GDOA simulation converges to three different solutions, close from each other. This can be explained by the fact that these three different EDP have the same performances ($\Delta G = 2.3$ dB and $G_{ave} = 18.8$ dB). It hence means that in the 4-D space of $\vec{\mathbf{X}}$, there is not a unique point that leads to the maximum of the function $F(\vec{\mathbf{X}})$, but a set of different points. In other words, several optimized EDP exist, allowing the same optimized amplifier performance. More precisely, the solution of the optimization is a tiny hyper volume in the solution space (meaning that the function $F(\vec{\mathbf{X}})$ is constant and equal to its maximum for a set of points that compose a continuous 4-D volume). Thus, the absolute maximum is obtained for several points, juxtaposed from each other and it is not possible to consider that one of them is a better solution than another.

The number of iterations is about 80, in order to be sure that the algorithm correctly converges to the solution. However, it can be seen that the most important stage of the convergence are the 20 first points and that the 40th point is already close to the solution. Note that the hypothesis made earlier on the validity and the suitability of the definition of the function F is validated: this function describes, as suited, the performances of the amplifier and ensures fast convergence toward the solution. On the gain figures [see Fig. 4(c)], it can be seen that centered modes (LP_{01} and LP_{02} signal modes) and off-centered modes

(LP_{11} and LP_{21} signal modes) do not present the same gain shape. This phenomenon is due, on one hand, to the fact that pump intensity is higher at the center of the core, while it is lower on the edges of the core, and on the other hand, to the difference of Er^{3+} concentration between the center and the periphery of the core. These two aspects lead to two different optimum fiber lengths for the modes. At the end of the fiber, off-centered modes are slightly more reabsorbed at 1530 nm compared to centered modes because of lower inversion of population at the periphery of the core. However, it can be seen that the difference of gain between the modes remains very low when compared to the spectral gain excursion of a given mode.

IV. Influence of the Pumping Scheme

In this section, pumping scheme will be used as a new DoF. Conventionally, ω_p could be considered as a new dimension for $\vec{\mathbf{X}}$, meaning that the GDOA will optimize this variable in order to get the optimum pumping scheme. However, we chose, in this section, to consider ω_p as a parameter, so as to make results more understandable and easier to analyze. Hence, for each pump waist from 1 to 6 μm, a GDOA has been used in order to find the EDP the best adapted for this particular pumping scheme. For each value of ω_p, the overlap integrals between the incident Gaussian pump beam and the guided modes at 980 nm—and, hence, the insertion losses—are calculated so that coupling of the pump beam in the active fiber is as realistic as possible.

Results are reported in Fig. 5. As can be seen, each modal composition at the pump wavelength implies different EDP. These results illustrate the close interdependance of EDP and pumping profile. Thus, in experiments, if pumping scheme is changed, the EDP should be adjusted to the new pumping scheme (i.e., the fiber should be replaced). Reciprocally, when using a particular fiber with optimized EDP, one should pay special attention to the way the pump beam is injected in the fiber, so as to get optimum performances.

For a wide range of ω_p values, high and flat gain is achievable (from 2 to 6 μm). However, with respect to performances [see Fig. 5(b)], high losses on pump injection have a strong impact on $G_{\rm ave}$. The lowest $\Delta G/G_{\rm ave}$ ratio is obtained for ω_p between 2 and 2.5 μm, and the EDP are equal to $[3.2, 3.8, 8, 8] \times 10^{24}$ ions.m^{-3} and $[3.8, 3.4, 7.9, 8] \times 10^{24}$ ions.m^{-3}, respectively.

Two particular solutions should be mentioned.

1) For ω_p close to 2.25 μm, a very simple EDP can be used, with only two different erbium concentrations. This profile is interesting in terms of fabrication complexity, since only two doping levels are needed, with clear differentiation between them (factor of 2 between the concentrations, approximately). This simple EDP is counterbalanced by a relatively complex pumping profile (equal excitation of LP_{01} and LP_{02} pump modes, with 80% of incident pump power coupled into the fiber core). Average gain is about 19 dB for a ΔG value of 2.5 dB ($\Delta G/G_{\rm ave} \simeq 0.13$).
2) For ω_p about 4.5 μm, pump power coupling efficiency is maximum and modal composition of the pump is essentially made of LP_{01} (99% of coupling efficiency). However, gain equalization is obtained *via* four different rod families in the fiber core. Average gain is about 21 dB and ΔG value is about 3.2 dB ($\Delta G/G_{\rm ave} \simeq 0.15$).

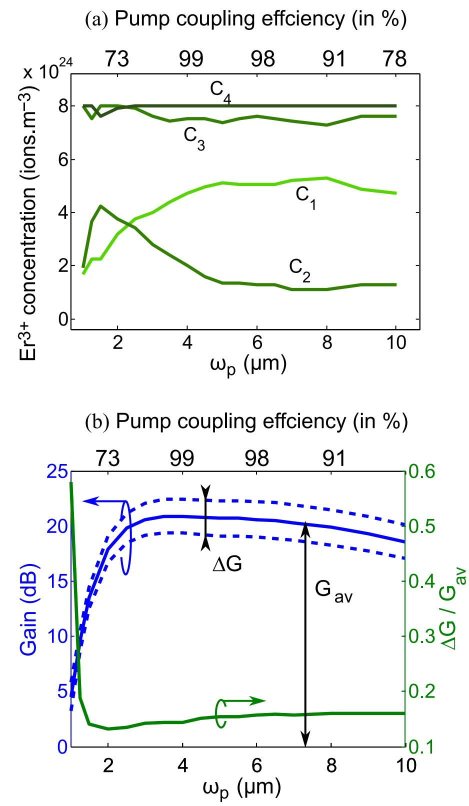

Fig. 5. (a) Optimum Er^{3+} concentration that can be found thanks to the GDOA, as a function of ω_p. (b) Performance of the optimized FM-EDFA as a function of ω_p.

V. Analysis of Particular Solutions

In this section, we will analyze the stability of the amplifier design as a function of signal and pump power for the two designs that were highlighted in the previous section.

First, we will consider the optimized amplifier corresponding to $\omega_p = 4.5$ μm and EDP made of four different erbium concentrations. In Fig. 6(a), a schematic of the EDP is reported. The amplifying performances as a function of pump power and signal power are reported in Fig. 6(b) and (c), respectively.

In Fig. 6(b), signal power has been fixed to -18 dBm per channel (i.e., -4 dBm total signal power) and fiber length has been fixed to the value that minimizes ΔG in each case, i.e., from 3.5 m (if $P_p = 100$ mW) to 9.2 m (if $P_p = 500$ mW). In Fig. 6(c), pump power has been fixed to 200 mW and fiber length has been adjusted from 4.6 m (if $P_s = 6$ dBm) to 9.6 m (if $P_s = -24$ dBm). One can see, in both figures, that the gain flatness ($\Delta G/G_{\rm ave}$) is minimum for $P_p = 200$ mW and $P_s = -4$ dBm (initial power conditions that were used to design the fiber). If power conditions are different than these initial values, gain equalization suffers from strong impairment (up to 0.2 for $\Delta G/G_{\rm ave}$ values). Thus, we can conclude that this particular

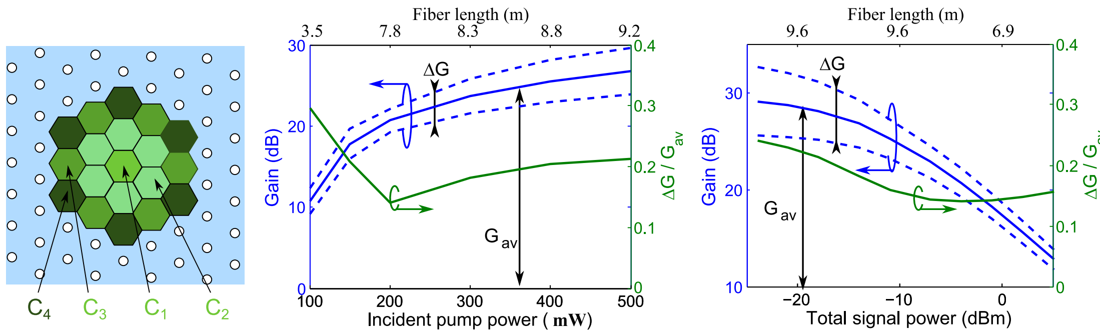

Fig. 6. (a) Schematic of the optimized erbium doping profile needed if $\omega_p = 4.5\ \mu$m: $C_1 = 5.0 \times 10^{24}$, $C_2 = 1.6 \times 10^{24}$, $C_3 = 7.5 \times 10^{24}$, and $C_4 = 8.0 \times 10^{24}$ ions.m^{-3}. Pump insertion losses are about 0.01 dB. (b) Performances of the optimized FM-EDFA as a function of incident pump power. Total signal power is fixed to -4 dBm. (c) Performances of the optimized FM-EDFA as a function of total signal power. Incident pump power is fixed to 200 mW.

amplifier design can be used only under particular signal and pump power conditions.

Now, we will focus on a design close to the optimum obtained for ω_p equal to 2.5 μm. We simplified the EDP to $[4,\ 4,\ 8,\ 8] \times 10^{24}$ ions.m^{-3} [see Fig. 7(a)] and ω_p is fixed to 2.4 μm for the following simulations. The stability of this design as a function of EDP, fiber length, pump power, and total signal power has been reported in Fig. 7(b)–(e), respectively.

In Fig. 7(d), signal power has been fixed to -4 dBm (i.e., -18 dBm per channel), and gain values have been computed as a function of pump power. $\Delta G/G_{\rm ave}$ ratio is relatively flat (about 0.14) for P_p from 200 to 500 mW [see Fig. 7(d)]. Note that fiber length has been adjusted for each pump power (from 4.2 m if $P_p = 100$ mW to 7.4 m if $P_p = 500$ mW).

As can be seen in Fig. 7(d), interesting performance can be obtained for $P_p = 300$ mW and $P_s = -4$ dBm (24 signal with -18 dBm each) with $G_{\rm ave} = 22.9$ dB and $\Delta G = 3.1$ dB, resulting on a $\Delta G/G_{\rm ave}$ ratio equal to 0.13 [cf., Fig. 7(f)]. In this particular case, noise figure (NF) values of the different channels are between 7.6 and 9.1 dB [cf., Fig. 7(g)].

Thus, pump power has been fixed to 300 mW [pump power with best flatness in Fig. 7(b)], and gain values have been computed as a function of total signal power [cf., Fig. 7(e)]. $\Delta G/G_{\rm ave}$ ratio is relatively flat (between 0.13 and 0.14) for total signal power from -18 dBm (24×-32 dBm) to 6 dBm (24×-8 dBm). Note that fiber length has been adjusted for each signal power level (from 10 m if $P_s = -24$ dBm to 4.5 m if $P_s = 6$ dBm).

It has to be noticed that this second amplifier design (design reported in Fig. 7, compared to the one of Fig. 6) has a fairly stable performance over a wide range of signal and pump power values. If we now focus our attention on the stability of this particular active fiber design, we can see in Fig. 7(b) that the optimum EDP has relatively good stability to erbium doping concentration deviation, which can be as high as 1×10^{24}, while gain flatness factor remains as low as 0.18 (12.5% of deviation for C_3 and C_4 and 25% of deviation for C_1 and C_2 for only 5% of degradation of gain flatness factor). The concentration C_2 seems to be the more sensitive, while C_3 seems to be the less sensitive. Concerning the stability of gain equalization as a function of fiber length, we can see in Fig. 7(c) that gain flatness factor is always below 0.16 from 6.5 to 8 m (interval representing 20% of optimal fiber length, for only 3% of gain flatness degradation).

Additional simulations were made considering more complex pumping scheme: three DoF were added to the GDOA in order to consider injection waist, but also offset injections in X and Y directions. These three additional DoF were optimized by GDOA so that pumping scheme was optimized at the same time as EDP. No better performances were found when compared to the designs reported in Figs. 6 and 7. So, it seems that off-centered pump injection is not relevant for the six-mode EDFA reported in the first part of this paper.

VI. From Six to Ten Modes FM-EDFA

In this last section, we focus the study on an amplifier design allowing gain equalization over C-band and over ten signal modes. The opto-geometrical parameters of the PCF have been adjusted to $\Lambda = 3.4\ \mu$m, and $d/\Lambda = 0.5$ so that the first 10 LP modes are guided in the fiber core at 1550 nm. Calculated intensity profiles of the guided modes in this fiber at 1550 and 980 nm are reported in Fig. 8. Then, GDOA has been used to find the best EDP and the best pumping profile that equalizes the gain. Initially, 10 DoF were used to find an optimized amplifier design.

1) 7 of the 10 DoF were dedicated to the EDP [as schematized in Fig. 3(b)].
2) 3 DoF were dedicated to the pumping scheme: one for pump injection waist (ω_p) and the two others for injection offset (in X and Y directions). For a fiber core with cylindrical symmetry, only one offset direction should be considered, but for an hexagonal-shaped core, the two transverse dimensions are not equivalent, and thus, should be considered separately.

The modal composition at the pump wavelength is obtained each time by computing the overlap integrals between the incident off-set Gaussian beam and the guided modes at 980 nm. Fiber length is adjusted at each iteration of the GDOA in order to minimize $\Delta G/G_{\rm ave}$ with the given EDP and given pumping

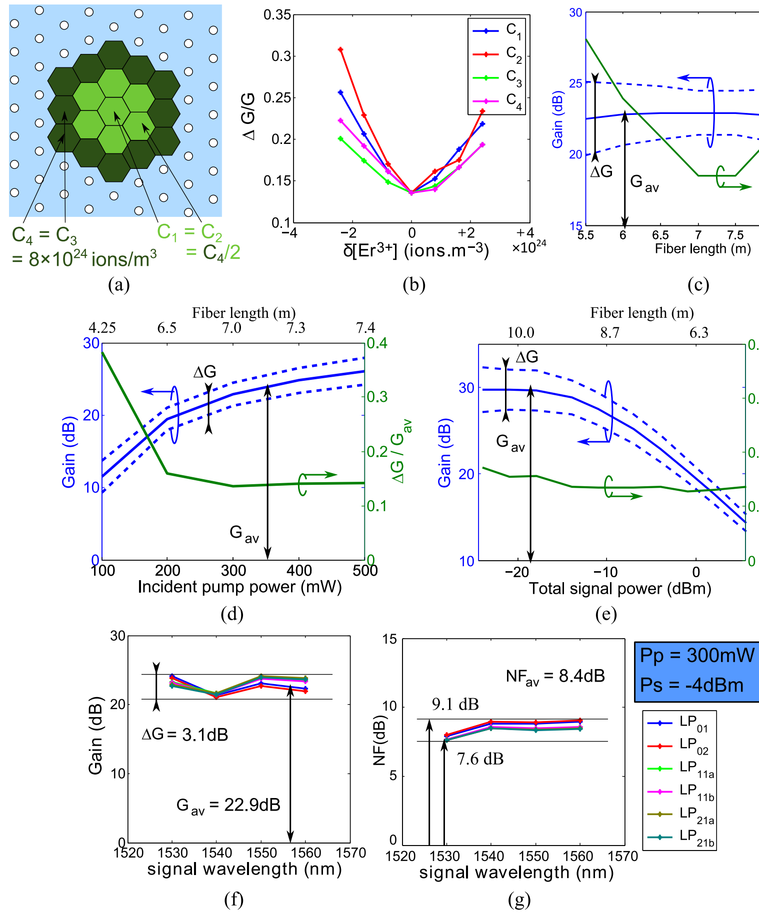

Fig. 7. (a) Simplest EDP that can be used to equalize the gain. Only two different erbium concentrations are used. (b) Study of stability of the EDP: evolution of gain flatness factor as a function of deviation to the optimum EDP in terms of erbium concentration. (c) Evolution of performances as a function of fiber length. d) Performances as a function of incident pump power. Total signal power is fixed to −4 dBm. (e) Performances of the optimized FM-EDFA as a function of total signal power. Incident pump power is fixed to 300 mW. (f) and (g) gain values and noise figure values for the 24 signals simultaneously amplified. Incident pump power is 300 mW (240 mW coupled into the fiber) and total signal power is −4 dBm.

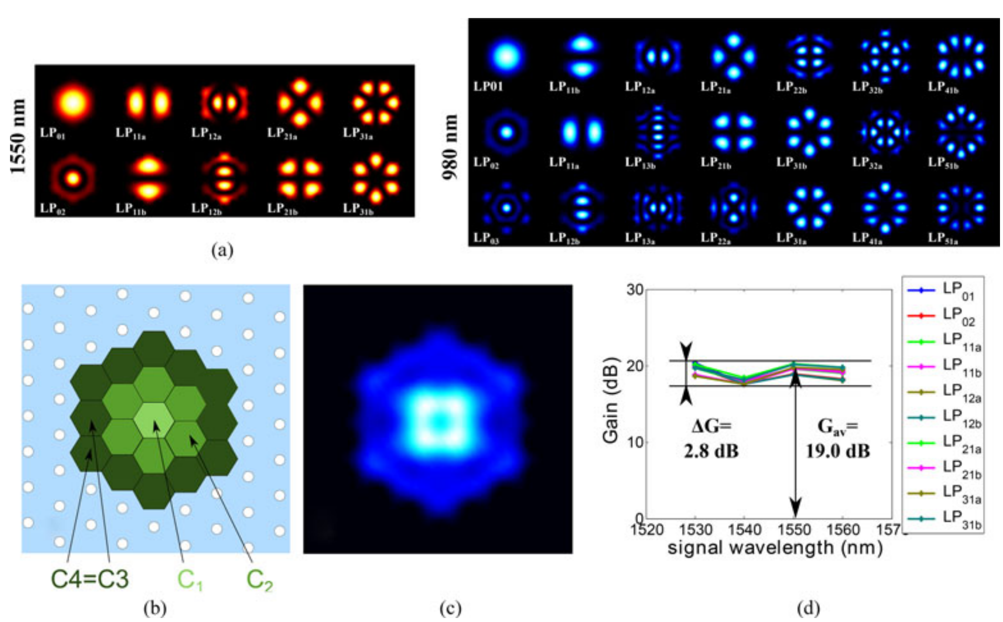

Fig. 8. (a) Intensity profiles of the guided modes, at 1550 and 980 nm, when $\Lambda = 3.4\ \mu$m, and $d/\Lambda = 0.5$ in Fig. 1(a). (b) Optimized erbium doping profile for ten-mode equalization: $C_1 = 3.4 \times 10^{24}$ ions.m^{-3}, $C_2 = 4.9 \times 10^{24}$ ions.m^{-3}, $C_3 = C_4 = 8 \times 10^{24}$ ions.m^{-3}. (c) Intensity pattern of the pump at the beginning of the EDF (i.e., z = 0), obtained with 1.5 μm-waist Gaussian beam, with 1.5 μm off-set injection for both X and Y axes. (d) Gain values over wavelengths and modes.

profile. Incident pump power is 200 mW and total signal power is equal to -4 dBm, distributed over 40 channels simultaneously amplified (4 wavelengths $\times$ 10 modes with -20 dBm per channel). Optimization through ten dimensions is not a straightforward problem, since several local solutions exist. The algorithm converges to different solutions that are not necessarily the real optimum. Thus, a large number of simulations has been performed, starting from different points, in order to have better chances to reach the best solution.

Finally, the best solution proposed by the GDOA leads to a core made of only three different rod families with $C_1 = 3.4\times 10^{24}$ ions.m^{-3}, $C_2 = 4.9\times 10^{24}$ ions.m^{-3}, $C_3 = C_4 = 8\times 10^{24}$ ions.m^{-3} [cf., Fig. 8(b)]. The pumping scheme associated to this geometry was found to be a 1.5 μm injection waist, with an injection offset of 1.5 μm along both X and X directions. With this particular pump injection condition, about 80% of the incident pump power is, hence, coupled into the fiber core with the following distribution: 16% of LP_{01}, 10% of LP_{02}, 5.4% of both LP_{11}, 10% of both LP_{12}, 3.4% of LP_{13b}, 3.9% of LP_{13a}, 2.4% of LP_{21b}, 7.1% of LP_{22b}, 2% of LP_{32b}, 1% of LP_{32a}, 1%LP_{51b}, and 1% of LP_{51a} pump modes. The intensity pattern of the pump has been plotted in Fig. 8(c). Optimal fiber length is found to be 5.2 m. With this particular amplifier design, average gain is 19.0 dB and gain excursion (ΔG) is 2.8 dB (gain flatness is 0.14) [cf., Fig. 8(d)]. NF values of the 40 signals were all between 8.0 and 9.4 dB. Thus, we can see that optimization algorithm as GDOA are fast and reliable tools assisting pretty well when designing FM-EDFAs. Note that further reduction of the gain excursion might be possible by considering more complex pumping scheme, and more complex erbium doping structure (for example, by including more pixels in the core).

VII. Conclusion

In this paper, a gradient descent optimization algorithm has been used to optimize the amplifying performances of an FM-EDF, so as to obtain similar gain shapes for several transverse modes over the C-band. Erbium doping profile and pumping scheme have been simultaneously optimized in the case of two fiber designs with micro-structured core supporting, respectively, six and ten spatial modes at the signal wavelength. The impact of pump insertion losses has also been studied and fiber designs offering gain excursion—over modes and wavelengths—below 15% have been isolated. These results show that an optimization algorithm allows to efficiently identify optimized amplifier designs. Such a work paves the way to the investigation of the impact of additional degrees of freedom: forward and/or backward pumping, fiber lengths in case of concatenation. All these parameters can be quickly optimized by using GDOA.

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